An Ant System with Direct Communication for the Capacitated Vehicle Routing Problem

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***Abstract*— Ant colony optimization (ACO) algorithms are population-based algorithms where ants communicate via their pheromone trails. Usually, this indirect communication leads the algorithm to a stagnation behaviour, where the ants follow the same path from early stages. This is because high levels of pheromone are generated into a single trail, where all the ants are influenced and follow it. As a result, the population gets trapped into a local optimum solution, which is difficult for the ACO algorithm to escape from it. In this paper, a direct communication scheme is proposed and applied to ACO for the capacitated vehicle routing problem (CVRP), which is a difficult *NP*-hard optimization problem. The direct communication scheme allows the ants to exchange customers from different routes, with other ants that belong to their communication range. Experiments show that the direct communication scheme significantly improves the solution quality of a conventional ACO algorithm regarding CVRP with and without service time constraints.**

I. Introduction

Ant colony optimization (ACO) algorithms are inspired from the behaviour of real ant colonies, when ants search for food from their nest to food sources. Ants cooperate and communicate indirectly via their pheromone, where they leave a trail to the path they explore. The more pheromone on a specific trail, the higher the possibility of that trail to be followed by the ants.

This behaviour inspired researchers to develop the first ACO algorithm, called the ant system (AS) [6], [8], which has been applied to the well-known travelling salesman problem (TSP). Moreover, AS has been applied to other combinatorial optimization problems, such as the quadratic assignment problem [10], the job scheduling problem [7], the vehicle routing problem (VRP) [2], [9], and many other optimization problems.

In this paper, we focus on the VRP because it shares many similarities with real-world applications [21], where a population of ants begins from a depot and visits customers (or delivery points). When the demand of all the customers are satisfied, the ants will return back to the depot. Each ant represents several vehicle routes, due to the vehicle capacity constraint, which represent a complete capacitated VRP (CVRP) solution. The AS has been applied to the CVRP [2], and later on improved [3] using different heuristic information. However, it suffers from the stagnation behaviour, where all ants generate the same solution from early iterations. This is because a high intensity of pheromones may be generated into a single trail, and the ants may stuck on a local optimum solution.

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In nature, ants do not only communicate indirectly by pheromone trails, but also directly with other ants and gather important information [15]. A direct communication (DC) scheme has been found beneficial to the TSP, where ants communicate and exchange cities [16]. In this paper, a similar DC scheme is proposed for ACO algorithms to address the CVRP. The ants will be able to communicate with other ants within their neighbourhood (or communication range), which is based on a similarity metric. Ants within their communication range are allowed to exchange customers with each other, only if there is an improvement and the CVRP constraints are not violated. Additionally, a small amount of pheromone is added to the exchanged customers in order to influence ants towards new promising paths generated from DC.

In order to investigate the performance of the proposed DC scheme for ACO algorithms, we consider the AS for the CVRP [3], and experiments are carried out to compare the conventional AS algorithm and the AS with the proposed DC scheme, denoted as AS+DC, on a set of benchmark CVRP instances. Experimental results show that the proposed scheme improves the solution quality of the conventional AS since it enables ants to avoid local optima and leads the population towards the global optimum.

The rest of the paper is organized as follows. Section II, defines the framework of two CVRP variations, i.e., the basic CVRP and CVRP with service time constraints. In Section III, we describe the AS algorithm for the CVRP, which will be used in the experiments. In Section IV, we describe the proposed DC scheme, giving details on how it can be applied to AS, and to any ACO algorithm. In addition, we discuss possible advantages and disadvantages of using this scheme. Section V presents the experimental results with the corresponding statistical tests of the proposed AS+DC in comparison with the conventional AS on different CVRP problem instances. Finally, Section VI provides concluding remarks and several directions for future work.

II. The Vehicle Routing Problem

The VRP became one of the most popular combinatorial optimization problems, due to its similarities with many real-world applications. The VRP is classified as *NP*-hard [13]. The basic VRP is the CVRP, where a number of vehicles with a fixed capacity need to satisfy the demand of all the customers, starting from and finishing to the depot. A VRP without the capacity constraint or with one vehicle can be seen as a TSP. There are many variations and extensions of the VRP, such as the multiple depot VRP, the VRP with pickup and delivery, the VRP with time windows and combinations of different variations (for more details see [23]). In this paper we will consider the CVRP with and without service time constraints.

Usually, the CVRP is represented by a complete weighted graph $G = (V, E)$, with $n + 1$ nodes, where $V = \{u_0, \ldots, u_n\}$ is a set of vertices corresponding to the customers (or delivery points) u_i ($i = 1, \cdots, n$) and the depot u_0 and $E = \{(u_i, u_j) : i \neq j\}$ is a set of edges. Each edge (u_i, u_j) is associated with a non-negative d_{ij} which represents the distance (or travel time) between u_i and u_j. For each customer u_i, a non-negative demand q_i is given for both the CVRPs with or without service time constraints, and, for the case of CVRP with service time constraints, an additional non-negative service time δ_i is given. For the depot u_0, a zero demand and service time is associated, i.e., $q_0 = \delta_0 = 0$.

The aim of the CVRP is to find the route (or a set of routes) with the lowest cost without violating the following constraints:

- Every customer is visited exactly once by only one vehicle.
- Every vehicle starts and finishes at the depot.
- The total demand of every vehicle route must not exceed the vehicle capacity Q.
- The total route travel time must not exceed the vehicle service time constraint L (in case of the CVRP with service time constraints).

A lot of algorithms have been proposed to solve small instances of different variations of the VRP, either exact or approximation algorithms [19], [22]. Although exact algorithms guarantee to provide the global optimum solution, an exponential time is required in the worst case scenario, because the CVRP is *NP*-hard [13]. On the other hand, approximation algorithms, i.e., evolutionary algorithms, can provide a good solution efficiently but cannot guarantee the global one [11].

In this paper, we focus on ACO algorithms due to their good results in real-world applications related to the VRP [21]. ACO algorithms are able to provide the optimum or a near-optimum solution in a sufficient amount of time, since they sacrifice their solution quality for the sake of efficiency (time) [18]. ACO algorithms have been successfully applied on the VRP with time windows [9], and to the VRP variation described in this paper [3] (for more details see Section III).

III. Conventional Ant System for the CVRP

A. Construction of Vehicle Routes

The AS has been applied to the CVRP [2], and later on improved using the savings algorithms [3], [5]. Each artificial ant will construct a complete CVRP solution, which consists of the routes of each vehicle. Initially, all the ants are placed on the depot. Each ant k uses a probabilistic rule to choose the next customer to visit. However, when the choice of the next customer would lead to infeasible solution, i.e., exceeding the maximum capacity or the total service time constraint of the vehicle, the depot is chosen and a new route is started. Therefore, the neighbourhood of available customers for ant k, when its current customer is u_i, is defined as $N_i^k = \{u_j \in V : u_j \text{ is feasible}\} \cup \{u_0\}$, and the probabilistic rule is defined as follows:

$$p_{ij}^k = \begin{cases} \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{u_l \in N_i^k} [\tau_{il}]^\alpha [\eta_{il}]^\beta}, & \text{if } u_j \in N_i^k, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where τ_{ij} is the existing pheromone trail between customers i and j, η_{ij} is the heuristic information available a priori, N_i^k denotes the neighbourhood of unvisited customers for ant k when its current customer is i, α and β are two parameters which determine the relative influence of τ and η, respectively, and they have a significant impact on ACO algorithms to achieve a robust behaviour. The heuristic information, i.e., η_{ij}, determines the visibility of customers and it is based on the parametrical saving function [3], which is defined as follows:

$$\eta_{ij} = d_{i0} + d_{0j} - g \times d_{ij} + f \times |d_{i0} - d_{j0}| \tag{2}$$

where d_{ij} is the distance (or travel time) between customers i and j, g and f are constant parameters.

B. Pheromone Trail Update

After all ants have visited all customers and generated a complete feasible CVRP solution, they update their pheromone trails using a rank-based method [1]. Initially, all trails contain an equal amount of pheromone on each edge. In each iteration, all the ants are ranked according to their solution quality. Only the $(\sigma - 1)$ best ranked ants, called elitist ants, and the best-so-far ant are allowed to deposit pheromone which is weighted according to their rank r. The best-so-far ant is weighted with the highest value, i.e., σ. Note that the best-so-far ant may not necessarily belong to the population of the current iteration. Then, the elitist ants and best-so-far ant retrace their solutions to deposit weighted pheromones to the edges that belong to their solutions. This process is defined as follows:

$$\tau_{ij} \leftarrow \tau_{ij} + \sum_{r=1}^{\sigma-1} (\sigma - r)\, \Delta\tau_{ij}^r + \sigma \Delta\tau_{ij}^{best}, \tag{3}$$

where r is the rank of an elitist ant, σ is the number of the elitist ants (including the best-so-far ant, denoted as $best$), and $\Delta\tau_{ij}^r$ is the amount of pheromone that the r-th best ant deposits, which is defined as follows:

$$\Delta\tau_{ij}^r = \begin{cases} 1/C^r, & \text{if } (u_i, u_j) \in T^r, \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

where T^r is the CVRP solution (or the vehicle routes) constructed by the r-th best ant and C^r is the total length of T^r. The corresponding $\Delta\tau_{ij}^{best}$ for the vehicle routes of the

Algorithm 1 AS for the CVRP

1: initialize data
2: **while** termination-condition not satisfied **do**
3: construct vehicle routes
4: update best ants
5: global pheromone update (evaporation + deposit)
6: **end while**

solution constructed by the best-so-far ant, is defined as in Eq. (4), but with C^{best} and T^{best}.

Furthermore, a constant amount of pheromone is deducted from all trails due to the evaporation of pheromone. This process enables the population to eliminate bad decisions from previous tours and is defined as follows:

$$\tau_{ij} \leftarrow (1-\rho)\,\tau_{ij},\ \forall\ (u_i, u_j), \tag{5}$$

where $0 < \rho \leq 1$ is the pheromone evaporation rate.

A general framework of a conventional ACO algorithm for the CVRP is represented in Algorithm 1.

IV. Direct Communication of Ants for the CVRP

The traditional AS algorithm suffers from the stagnation behaviour, where all ants follow the same path from the initial stages of the algorithm. This is because a high intensity of pheromone is generated to a single trail and attracts the ants to those areas. Therefore, AS is more likely to get trapped in a local optimum solution, which may degrade the solution quality.

In nature, ant colonies communicate not only indirectly via their pheromone trails, but also directly by exchanging important information [15]. To avoid the stagnation behaviour, we can integrate a DC scheme into AS algorithms by allowing ants to exchange customers after they construct their vehicle routes, as shown in Algorithm 2. The DC scheme is based on adaptive swaps and has been recently applied to the TSP with promising results [16]. For the CVRP, a similar scheme is proposed, where each ant ant_k communicates with another ant within its communication range as follows:

1) A customer u_i is randomly selected from ant ant_k.
2) The successor and predecessor of u_i, i.e., customers u_{i-1} and u_{i+1}, respectively, are selected from ant_k.
3) Another ant is selected, denoted ant_j, from the communication range of ant_k and customer u_i is located in ant_j.
4) The successor and predecessor of u_i, i.e., customers u'_{i-1} and u'_{i+1}, respectively, are selected from ant_j and located in ant_k.
5) Swaps are performed in ant_k between customers u_{i-1} and u'_{i-1} and between customers u_{i+1} and u'_{i+1}.
6) A small extra amount of pheromone is deposited to the resulting edges between u_i and its successor and between u_i and its predecessor in ant_k.

The proposed communication scheme has a high risk to degrade the solution quality of the tours constructed by the ants and disturb the optimization process. Therefore, only

Algorithm 2 AS+DC for the CVRP

1: initialize data
2: **while** termination-condition not satisfied **do**
3: construct vehicle routes
4: perform direct communication
5: update best ants
6: global pheromone update (evaporation + deposit)
7: local pheromone update
8: **end while**

the swaps which are beneficial are allowed in order to limit the risk. For example, if d_{ij} between the current successor city of city u_i in ant ant_k is less than the successor city obtained from the neighbour ant ant_j, then ant_k remains unchanged. The same happens with the predecessor city of city u_i in ant ant_k. Moreover, the swaps may violate the capacity and service time constrains of the vehicle routes and generate an infeasible solution. Therefore, only the swaps that do not violate any of the constraints are allowed. The swap method for the CVRP has been introduced in [19], where customers from different vehicle routes are exchanged. In our case, the moves of the swaps are made adaptively since they are inherited from other ants.

Apart from the swaps, a small amount of pheromone is deposited to the edges affected by the adaptive swaps in order to determine the influence and explore possible improvements. This process is defined as follows:

$$\tau_{ij} \leftarrow \tau_{ij} + \frac{(\tau_{max} - \tau_{min})(1-w)}{n}, \tag{6}$$

where w is a constant parameter to determine the influence of the possible improvements and n is the number of customers.

The communication range of an ant ant_k with other ants is based on the similarities of ants and is defined as follows:

$$R_k = \{ant_j \in P | 1 - \frac{CE_{kj}}{n + avg(NV_k, NV_j)} \leq T_r\}, \tag{7}$$

where P is the population of ants, T_r is a predefined threshold which determines the size of the communication range of an ant, n is the number of customers, CE_{kj} are the common edges between two ants, i.e., ant_k and ant_j, and NV_k and NV_j are the number of vehicles ant_k and ant_j have in their solutions, respectively. If an edge (u_l, u_m) or (u_m, u_l) that appears in the solution of ant_k also appears in the solution of ant_j, then it is counted as a common edge between ant_k and ant_j. A larger value of T_r indicates a larger communication neighbourhood of dissimilar ants, whereas a value closer to 0 indicates a smaller communication neighbourhood of similar or identical ants.

The DC scheme can be applied to any ACO algorithm right after all ants construct their solutions and before they update their pheromone trails globally. Note that the exchange of customers is performed locally and customers are exchanged when there is an improvement on the distance of the edge. In addition, the edge receives an extra amount of pheromone locally, as in Eq. (6), to attract ants to perform more exploration on possible promising areas in the search space. Therefore, the newly discovered areas on the search space will be considered by the ants in the next iterations.

Moreover, the swaps may reduce the number of vehicles used, which usually leads to better solution quality for the CVRP. For example, ant_k has the following solution:

$\{$(0,1,4,2,0), (0,3,5,6,0), (0,7,9,0), (0,10,0)$\}$,

which consists of 4 vehicles, and communicates with ant_j with the following solution:

$\{$(0,5,3,1,0), (0,6,2,10,0), (0,9,7,4,8,0)$\}$,

which consists of 3 vehicles. The customer which is randomly selected from ant_k is customer 2 with customer 4 and depot 0 as the predecessor and successor, respectively. Then, customer 2 is located in ant_j (selected from ant_k's communication range), and obtains customers 6 and 10, which are the predecessor and successor of customer 2 in ant_j, respectively. Assume that the swaps of customers $(4, 6)$ and $(0, 10)$ satisfy the capacity and service time constraints and there is an improvement on the distance. The resulting solution of ant'_k is:

$\{$(0,1,6,2,10,0), (0,3,5,4,0), (0,7,9,0), (0,0,0)$\}$

where the last route is removed since it does not contain any customers, and, thus, the solution consists of 3 vehicles.

Moreover, it has been shown that the solution quality of the routes constructed from the ants can be significantly improved with the use of a local search operator [14]. However, such methods may increase the computation time significantly especially on large problem instances. In our experiments, we will not consider any local search operator in order to investigate the effect of the proposed scheme, but it is worth to be considered for future work.

The aim of DC scheme is to exchange customers from different vehicles routes or from the same vehicle route, and take advantage of different solutions constructed by ants on each iteration. It is possible that the solution of an ant may be worse than the best ant, but a sub-tour may belong to the global optimum. It is also possible that a sub-tour in the best tour may belong to a local optimum. Therefore, it is difficult for an ACO algorithm to escape from local optimum because the pheromone trails will always lead the ants into the same path. The adaptive swaps may help to eliminate such behaviour and possibly enhance the solution quality of the conventional AS algorithm.

V. Experimental Study

A. Experimental Setup

The proposed AS+DC is compared with the conventional AS [3], which is the only existing ACO algorithm for the CVRP. For the experiments, 14 CVRP instances with and without service time constraints are considered as described in [4]. The problem instances contain between 50 and 199 customers in addition to the single depot. The problem instances $C1-C10$ are randomly distributed, whereas the problem instances $C11-C14$ are clustered. The problem instances $C1-C5$ and $C11-C12$ are identical with the problem instances $C6-C10$ and $C13-14$, respectively, expect that the latter have service time constraints. For such problem instances, all the customers have the same service time $\delta = \delta_1 = \ldots = \delta_n$.

TABLE I

The problem characteristics and mean results of the best solution averaged over 30 runs for the CVRP without service time constraints

Inst.	n	Q	L	δ	AS	AS+DC	Sign.
Random Problems							
C1	50	160	∞	0	639.43	626.90	$+$
C2	75	140	∞	0	1062.33	1045.40	$+$
C3	100	200	∞	0	1214.06	1145.70	$+$
C4	150	200	∞	0	1751.73	1668.26	$+$
C5	199	200	∞	0	2321.20	2203.73	$+$
Clustered Problems							
C11	120	200	∞	0	1589.96	1524.83	$+$
C12	100	200	∞	0	1223.03	1170.90	$+$

TABLE II

The problem characteristics and mean results of the best solution averaged over 30 runs for the CVRP with service time constraints

Inst.	n	Q	L	δ	AS	AS+DC	Sign.
Random Problems							
C6	50	160	200	10	647.33	624.50	$+$
C7	75	140	160	10	1055.76	1047.76	$\sim$
C8	100	200	230	10	1188.53	1145.56	$+$
C9	150	200	200	10	1743.86	1655.83	$+$
C10	199	200	200	10	2320.36	2182.73	$+$
Clustered Problems							
C13	120	200	720	50	1749.23	1722.13	$+$
C14	100	200	1040	90	1200.60	1179.26	$+$

For each experiment on each problem instance, 30 independent runs of each algorithm were performed for statistical purposes. For each run, 1000 iterations were performed in order to have the same number of evaluations (expect on problem instances $C5$ and $C10$ where 2500 iterations were performed) and an observation of the best-so-far ant was taken every iteration.

B. Parameter Setting

Most of the parameters used in the algorithms are inspired from the literature since they have been found effective [3], [16]. For both AS and AS+DC, the parameters α and β used in Eq. (1) were set to 1 and 5, respectively. The evaporation constant ρ used in Eq. (5) was set to 0.75 as in [3]. However, adapting the evaporation rate may help the population escape possible local optima and may improve the overall performance of ACO algorithm and it worth future investigation. The constant parameters $f = g = 2$ in Eq. (2) and $\sigma = 6$ in Eq. (3). The population size was set to $m = 50$ for both algorithms. A good value for the w parameter used

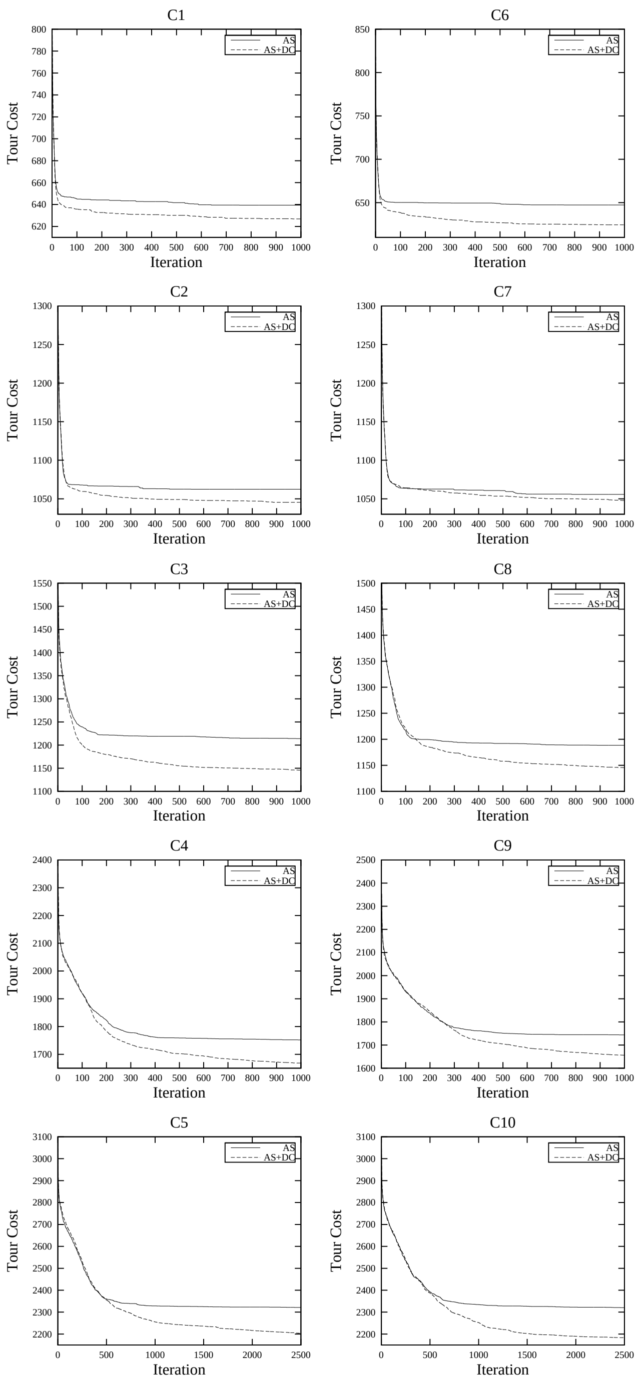

Fig. 1. Dynamic performance (averaged over 30 runs) of the AS algorithm with and without DC for the random CVRP problem instances.

in Eq. (6) was found to be 0.5 and $T^r = 0.7$ in Eq. (7) from our preliminary experiments.

C. Analysis of the Results

In Tables I and II, the characteristics of the problem instances and mean results of AS and AS+DC are presented for the CVRPs without and with service time constraints, respectively. In Tables I and II, the corresponding statistical results of comparing the AS+DC and AS algorithms by a

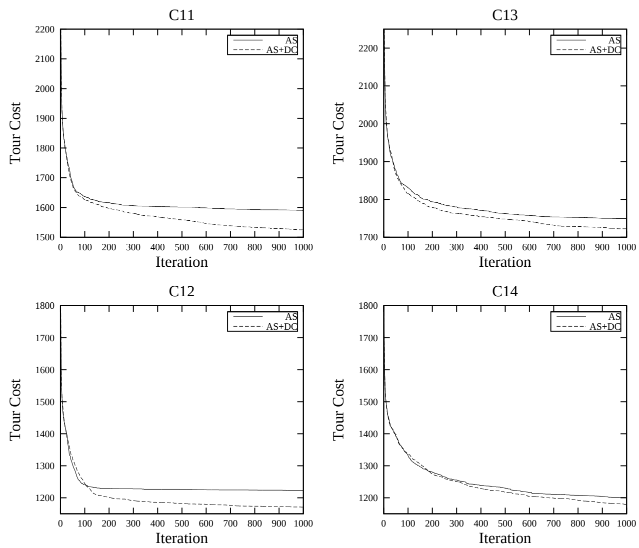

Fig. 2. Dynamic performance (averaged over 30 runs) of the AS algorithm with and without DC for the clustered CVRP problem instances.

non-parametric statistical test, i.e., Wilcoxon rank-sum test, at the 0.05 level of significance are also shown in the last column, where "$+$" indicates that the AS+DC algorithm is significantly better than the AS algorithm, and "$\sim$" indicates no statistical significance. Furthermore, in Figs. 1 and 2, the best-so-far solution, averaged over 30 runs, of the algorithms over each iteration are plotted, for the random and clustered problem instances, respectively.

In both CVRP variations, the AS+DC algorithm outperforms the conventional AS algorithm on almost all the problem instances, which can be observed from Tables I and II, expect on $C7$ where they are not significantly different. This is because the conventional AS algorithm has more chances to get trapped on local optimum due to the stagnation behaviour.

On the other hand, the proposed AS+DC algorithm increases the exploration ability and avoids possible local optima. This can be observed from Figs. 1 and 2, where the two algorithms have similar convergence in most of the problem instances, but the AS algorithm gets trapped to a local optimum solution, while the AS+DC algorithm keeps exploring the search space and improves the solution quality. This is due to the extra local pheromone update of the DC scheme to the possible improvements found from the adaptive swaps. Moreover, in some cases, i.e., $C8$, $C12$, and $C14$ the convergence of AS+DC is slightly delayed.

However, when the population of ants is identical, the DC scheme will not be effective since the ants will communicate with identical ants. This may be a possible reason why the AS+DC algorithm is not significantly better in some cases, e.g., on $C7$.

VI. Conclusions

The communication between ants in conventional ACO algorithms is achieved indirectly via pheromone trails. In this paper, a scheme is proposed for ACO to solve the CVRP with and without service time constraints, which enables the ants to communicate both directly and indirectly. The ants are allowed to communicate and exchange customers from different vehicle routes, using adaptive swaps. Although one solution may not be better than another, it may contain a sub-tour which corresponds to the global optimum. The DC scheme helps ACO algorithms to escape from local optimum solutions and hence improves the solution quality. However, when the stagnation behaviour is reached, it becomes ineffective because all the ants in the population will be identical.

For the experiments, we use the AS algorithm, which is the only ACO algorithm applied to the CVRP, with or without the proposed DC scheme, on a set of CVRP instances. Generally speaking, the experimental results show clearly that the use of the proposed scheme with an appropriate communication range between ants improves the overall performance of the AS algorithm for the CVRP. The improvement regarding the solution quality is significant on almost all problem instances.

For further work, it will be interesting to apply the DC scheme with other ACO algorithms for other variations of VRPs [21]. Another future work is to investigate the effect of DC when an ACO algorithm is applied with a local search operator, e.g., the 2-opt operator [14]. Usually, on larger problem instances, the solution quality of algorithms is more likely to be degraded, whereas a local search may improve it at the price of more computation time. Therefore, DC may be able to guide the local search operator for better solution quality and computation time. Finally, the proposed approach may be effective for VRPs under dynamic environments since it delays the convergence and provides valuable diversity to the population of ACO algorithms [17]. Such characteristics are suitable to help the population to adapt well to environmental changes [12].

References

[1] B. Bullnheimer, R. F. Hartl, and C. Strauss, "A new rank based version of the ant system - a computational study," *Central European Journal for Operations Research and Economics*, vol. 7, no. 1, pp. 25–38, 1999.

[2] B. Bullnheimer, R. F. Hartl, and C. Strauss, "Applying the ant system to the vehicle routing problem," in *Meta-Heuristics: Advances and Tends in Local Search Paradigms for Optimization*, S. Voss et al., Eds., 1999, pp. 285–296.

[3] B. Bullnheimer, R. F. Hartl, and C. Strauss, "An improved ant system algorithm for the vehicle routing problem," *Annals of Operations Research*, vol. 89, pp. 319–328, 1999.

[4] N. Christofides, A. Mingozzi and P. Toth, "The vehicle routing problem," in *Combinatorial Optimization*, N. Christofides, A. Mingozzi, P. Toth and C. Sand, Eds., 1979, pp. 315-338.

[5] G. Clarke and J. W. Wright, "Scheduling of vehicles from central depot to a number of delivery points," *Operations Research*, vol. 12, pp. 568–581, 1964.

[6] A. Colorni, M. Dorigo, and V. Maniezzo, "Distributed optimization by ant colonies," in *Proc. of the 1st European Conf. on Artificial Life*, 1992, pp. 134–142.

[7] A. Colorni, M. Dorigo, V. Maniezzo, and M. Trubian, "Ant system for job-shop scheduling," *Belgian Journal of Operational Research*, vol. 34, pp. 39–53, 1994.

[8] M. Dorigo, V. Maniezzo, and A. Colorni, "Ant system: optimization by a colony of cooperating agents," *IEEE Trans. on Systems, Man and Cybernetics – Part B: Cybernetics*, vol. 26, no. 1, pp. 29–41, 1996.

[9] L. M. Gambardella, E. Taillard, and G. Agazzi, "MACS-VRPTW: a multiple ant colony system for vehicle routing problems with time windows," in *New ideas in optimization*, D. Corne, M. Dorigo, and F. Glover, Eds., 1999, pp. 63–76.

[10] L. M. Gambardella, E. Taillard and M. Dorigo, "Ant colonies for the quadratic assignment problems," *Journal of Operations Research Society*, vol. 50, pp. 167–176, 1999.

[11] J. He and X. Yao, "From an individual to a population: An analysis of the first hitting time of population-based evolutionary algorithms," *IEEE Trans. on Evolutionary Computation*, vol. 6, no. 5, pp. 495–511, 2002.

[12] Y. Jin and J. Branke, "Evolutionary optimization in uncertain environments - a survey," *IEEE Trans. on Evolutionary Computation*, vol. 9, no. 3, pp. 303–317, 2005.

[13] M. Labbe, G. Laporte, and H. Mercure, "Capacitated vehicle routing on trees," *Operations Research*, vol. 39, no. 4, pp. 61–622, 1991.

[14] S. Lin and B. W. Kerninghan, "An effective heuristic algorithm for the traveling salesman problem," *Operations Research*, vol. 21, no. 2, pp. 498–516, 1973.

[15] Y. Li and S. Gong, "Dynamic ant colony optimization for TSP," *International journal of advanced manufacturing technology*, vol. 22, no. 7-8, pp. 528–533, 2003.

[16] M. Mavrovouniotis and S. Yang, "Ant colony optimization with direct communication for the traveling salesman problem," in *Proc. of the 2010 UK Workshop on Computational Intelligence (UKCI2010)*, 2010.

[17] R. Montemanni, L. M. Gambardella, A. E. Rizzoli, and A. V. Donati, "Ant colony system for a dynamic vehicle routing problem," *Journal of Combinatorial Optimization*, vol. 10, pp. 327–343, 2005.

[18] F. Neumann and C. Witt, "Runtime analysis of a simple ant colony optimization algorithm," *Algorithmica*, vol. 54, no. 2, pp. 243–255, 2009.

[19] I. Osman, "Metastrategy simulated annealing and tabu search algorithms for the vehicle routing problem," *Annals of Operations Research*, vol. 41, pp. 421-451, 1993.

[20] M. Reimann, M. Stummer, and K. Doerner, "A savings based ant system for the vehicle routing problem," in *Proc. of the Genetic and Evolutionary Computation Conference (GECCO 2002)*, W. B. Langdon et al., Eds., pp. 1371–1325, 2002.

[21] A. E. Rizzoli, R. Montemanni, E. Lucibello, and L. M. Gambardella, "Ant colony optimization for real-world vehicle routing problems: from theory to applications," *Swarm Intelligence*, vol. 1, pp. 135–151, 2007.

[22] P. Toth and D. Vigo, "Branch-and-bound algorithms for the capacitated VRP," in *The Vehicle Routing Problem*, P. Toth and D. Vigo, Eds., pp. 2951, 2001.

[23] P. Toth and D. Vigo, "An overview of vehicle routing problems," in *The Vehicle Routing Problem*, P. Toth and D. Vigo, Eds., pp. 126, 2001.